

Exercise 13

If a ball is thrown into the air with a velocity of 40 ft/s, its height (in feet) after t seconds is given by $y = 40t - 16t^2$. Find the velocity when $t = 2$.

Solution

Determine the velocity first.

$$\begin{aligned}v(t) &= y'(t) \\&= \lim_{h \rightarrow 0} \frac{y(t+h) - y(t)}{h} \\&= \lim_{h \rightarrow 0} \frac{[40(t+h) - 16(t+h)^2] - [40t - 16t^2]}{h} \\&= \lim_{h \rightarrow 0} \frac{[40t + 40h - 16(t^2 + 2th + h^2)] - 40t + 16t^2}{h} \\&= \lim_{h \rightarrow 0} \frac{(40t + 40h - 16t^2 - 32th - 16h^2) - 40t + 16t^2}{h} \\&= \lim_{h \rightarrow 0} \frac{40h - 32th - 16h^2}{h} \\&= \lim_{h \rightarrow 0} (40 - 32t - 16h) \\&= 40 - 32t\end{aligned}$$

Therefore, the velocity at $t = 2$ is

$$v(2) = 40 - 32(2) = -24 \frac{\text{ft}}{\text{s}}.$$